B. Sc. Part III (Hons.) 6^{th} Paper

Groups

Direct product of two groups

External | Internal

I. External Direct product

Let G_1 and G_2 be any two groups, the binary operation in each being denoted "multiplicatively".

Then $G_1 \times G_2 = \{(g_1, g_2) : g_1 \in G_1, g_2 \in G_2\}$

Now, we define a binary operation on $G_1 \times G_2$ denoted multiplicatively as follows:

$(g_1, g_2)(h_1, h_2) = (g_1 h_1, g_2 h_2)$ where

$g_1, h_1 \in G_1 ; \; g_2, h_2 \in G_2$

For this binary operation, $G_1 \times G_2$ is a group. This group is known as External direct product of G_1 by G_2 or External direct product group of G_1 and G_2.

Another definition by Gallian

Let $G_1, G_2, \ldots, G_n$ be a finite collection of groups. The external direct product of $G_1, G_2, \ldots, G_n$, written as $G_1 \oplus G_2 \oplus \cdots \oplus G_n$ is the set of all n-tuples for which the ith component is an element of G_i and the operation is componentwise.

Symbolically,

$$G_1 \oplus G_2 \oplus \cdots \oplus G_n = \{(g_1, g_2, \ldots, g_n) : g_i \in G_i\}$$

where $(g_1, g_2, \ldots, g_n)(g_1', g_2', \ldots, g_n')$ is defined to be $(g_1 g_1', g_2 g_2', \ldots, g_n g_n')$.

Each product $g_i g_i'$ is performed with the operation of G_i.